

Calculus I Practice Final — Worked Answer Key

A balanced twenty-five-question cumulative final with point values, study map, and complete solutions.

Revision: 2026-07-23 **Canonical page:** <https://bettergrades.net/subjects/math/calculus/practice-exams/calculus-1-final/>

1. Evaluate $\lim_{x \rightarrow 2}(x^2 + 3x - 1)$.

Answer: 9

- The polynomial is continuous, so substitution gives $2^2 + 3(2) - 1 = 4 + 6 - 1 = 9$.
- The verified result is 9.

2. Evaluate $\lim_{x \rightarrow 4}(x^2 - 16)/(x - 4)$.

Answer: 8

- For $x \neq 4$, $(x^2 - 16)/(x - 4) = x + 4$; therefore the limit is $4 + 4 = 8$.
- The verified result is 8.

3. Choose k so $f(x) = kx + 1$ for $x < 2$ and $f(x) = 7$ for $x \geq 2$ is continuous.

Answer: $k = 3$

- Continuity requires $2k + 1 = 7$, so $2k = 6$ and $k = 3$.
- The verified result is $k = 3$.

4. Use the limit definition to find the derivative of x squared.

Answer: $2x$

- $f'(x) = \lim_{h \rightarrow 0}((x + h)^2 - x^2)/h = \lim_{h \rightarrow 0}(2x + h) = 2x$.
- The verified result is $2x$.

5. Differentiate $4x^5 - 3x^2 + 7$.

Answer: $20x^4 - 6x$

- $d(4x^5)/dx = 20x^4$, $d(-3x^2)/dx = -6x$, and the constant derivative is zero.
- The verified result is $20x^4 - 6x$.

6. Differentiate $x^2 \sin x$.

Answer: $2x \sin x + x^2 \cos x$

- The product rule gives $(x^2)' \sin x + x^2(\sin x)' = 2x \sin x + x^2 \cos x$.
- The verified result is $2x \sin x + x^2 \cos x$.

7. Differentiate $(x + 1)/(x - 1)$.

Answer: $-2/(x - 1)^2$

- The quotient rule gives $((x - 1) - (x + 1))/(x - 1)^2 = -2/(x - 1)^2$, for $x \neq 1$.
- The verified result is $-2/(x - 1)^2$.

8. Differentiate $(3x^2 + 1)^4$.

Answer: $24x(3x^2 + 1)^3$

- With $u = 3x^2 + 1$, $d(u^4)/dx = 4u^3(6x) = 24x(3x^2 + 1)^3$.
- The verified result is $24x(3x^2 + 1)^3$.

9. For $x^2 + y^2 = 25$, find dy/dx .

Answer: $-x/y$

- Differentiating gives $2x + 2y dy/dx = 0$, hence $dy/dx = -x/y$ where $y \neq 0$.
- The verified result is $-x/y$.

10. Find the tangent line to $y = x^3$ at $x = 2$.

Answer: $y - 8 = 12(x - 2)$

- At $x = 2$, $y = 8$ and $y' = 3x^2 = 12$, so point-slope form is $y - 8 = 12(x - 2)$.
- The verified result is $y - 8 = 12(x - 2)$.

11. Find c guaranteed by MVT for $f(x) = x^3$ on $[1, 3]$.

Answer: $c = 2$

- The secant slope is $(9 - 1)/(3 - 1) = 4$; solving $f'(c) = 2c = 4$ gives $c = 2 \in (1, 3)$.
- The verified result is $c = 2$.

12. Find critical numbers of $x^3 - 3x$.

Answer: $x = \pm 1$

- $f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$, so the critical numbers are $x = -1, 1$.
- The verified result is $x = \pm 1$.

13. Where is $x^3 - 3x$ increasing?

Answer: $(-\infty, -1) \cup (1, \infty)$

- Since $f'(x) = 3(x^2 - 1) > 0$ exactly when $|x| > 1$, the function increases on $(-\infty, -1) \cup (1, \infty)$.
- The verified result is $(-\infty, -1) \cup (1, \infty)$.

14. Find the inflection point of $x^3 - 6x^2$.

Answer: $(2, -16)$

- $f''(x) = 6x - 12$ changes from negative to positive at $x = 2$, and $f(2) = 8 - 24 = -16$.
- The verified result is $(2, -16)$.

15. A rectangle has perimeter 40. Find maximum area.

Answer: 100

- If the sides are x and $20 - x$, then $A = x(20 - x)$; $A' = 20 - 2x = 0$ at $x = 10$, giving $A = 100$.
- The verified result is 100.

16. A circle radius grows at 2 cm/s. Find dA/dt at $r=5$.

Answer: $20\pi \text{ cm}^2/\text{s}$

- From $A = \pi r^2$, $dA/dt = 2\pi r dr/dt = 2\pi(5)(2) = 20\pi \text{ cm}^2/\text{s}$.
- The verified result is $20\pi \text{ cm}^2/\text{s}$.

17. Use linearization at 9 to approximate $\sqrt{9.2}$.

Answer: $3 + 0.2/6 \approx 3.0333$

- For $f(x) = \sqrt{x}$, $f(9) = 3$ and $f'(9) = 1/6$, so $L(9.2) = 3 + 0.2/6 \approx 3.0333$.
- The verified result is $3 + 0.2/6 \approx 3.0333$.

18. Find $\int (6x^2 - 4)dx$.

Answer: $2x^3 - 4x + C$

- Termwise integration gives $6x^3/3 - 4x + C = 2x^3 - 4x + C$.
- The verified result is $2x^3 - 4x + C$.

19. Evaluate $\int_0^2 3x^2 dx$.

Answer: 8

- An antiderivative is x^3 , so $[x^3]_0^2 = 8 - 0 = 8$.
- The verified result is 8.

20. Differentiate $F(x) = \int_1^x \cos(t^2)dt$.

Answer: $\cos(x^2)$

- FTC Part I evaluates the integrand at the variable upper bound: $F'(x) = \cos(x^2)$.
- The verified result is $\cos(x^2)$.

21. Evaluate $\int 2x(x^2 + 1)^3 dx$.

Answer: $(x^2 + 1)^4/4 + C$

- Let $u = x^2 + 1$, so $du = 2x dx$; then $\int u^3 du = u^4/4 + C = (x^2 + 1)^4/4 + C$.
- The verified result is $(x^2 + 1)^4/4 + C$.

22. Find the area under $y=x$ on $[0,3]$.

Answer: $9/2$

- $A = \int_0^3 x dx = [x^2/2]_0^3 = 9/2$.
- The verified result is $9/2$.

23. Find the average value of x squared on $[0,3]$.

Answer: 3

- $f_{\text{avg}} = \frac{1}{3-0} \int_0^3 x^2 dx = \frac{1}{3} [x^3/3]_0^3 = 3$.
- The verified result is 3.

24. If $v(t)=3t^2-6t$, find displacement from 0 to 3.

Answer: 0

a. $\int_0^3 (3t^2 - 6t) dt = [t^3 - 3t^2]_0^3 = (27 - 27) - 0 = 0.$

b. The verified result is 0.

25. Explain why a differentiable function is continuous.

Answer: The derivative limit forces $f(x) - f(a)$ to approach 0.

a. Write $f(x) - f(a) = ((f(x) - f(a))/(x - a))(x - a)$; differentiability makes the first factor approach $f'(a)$ while the second approaches zero.

b. The verified result is The derivative limit forces $f(x) - f(a)$ to approach 0..