Calculus II Practice Final — Worked Answer Key

A balanced twenty-five-question Calculus II final with study map and complete worked key.

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1. Evaluate $\int xe^x dx$.

   Answer: $e^x(x-1)+C$

   a. With $u=x$, $dv=e^x dx$, $I = xe^x - \int e^x dx = e^x(x-1)+C$.

   b. The verified result is $e^x(x-1)+C$.

2. Evaluate $\int \sin^3 x \cos x dx$.

   Answer: $\sin^4 x/4 + C$

   a. Let $u=\sin x$, $du=\cos x dx$; then $\int u^3 du = u^4/4 + C = \sin^4 x/4 + C$.

   b. The verified result is $\sin^4 x/4 + C$.

3. Evaluate $\int 1/(x^2-1) dx$.

   Answer: $\frac{1}{2}\ln|\frac{x-1}{x+1}| + C$

   a. $1/(x^2-1) = \frac{1}{2}/(x-1) - \frac{1}{2}/(x+1)$; integrating gives $\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$.

   b. The verified result is $\frac{1}{2}\ln|\frac{x-1}{x+1}| + C$.

4. Determine convergence of $\int_1^\infty x^{-2} dx$.

   Answer: converges to 1

   a. $\lim_{b\to\infty}[-x^{-1}]_1^b = \lim_{b\to\infty}(1-1/b) = 1$, so the improper integral converges.

   b. The verified result is converges to 1.

5. Find area between y=x and y=x squared on [0,1].

   Answer: 1/6

   a. On $[0,1]$, $x \geq x^2$, so $A = \int_0^1 (x-x^2)dx = [x^2/2 - x^3/3]_0^1 = 1/6$.

   b. The verified result is 1/6.

6. Rotate y=x on [0,2] about x-axis. Find volume.

   Answer: $8\pi/3$

   a. $V = \pi \int_0^2 x^2 dx = \pi[x^3/3]_0^2 = 8\pi/3$.

   b. The verified result is $8\pi/3$.

7. A force F(x)=4x acts from x=0 to 3. Find work.

   Answer: 18

   a. $W = \int_0^3 4x\, dx = [2x^2]_0^3 = 18$.

   b. The verified result is 18.

8. Find $\lim_{n\to\infty} n/(n+1)$.

   Answer: 1

   a. Divide by n: $n/(n+1) = 1/(1+1/n) \to 1$.

   b. The verified result is 1.

9. Sum $\sum_{n=0}^{\infty} 3(1/2)^n$.

   Answer: 6

   a. Here $a = 3$, $r = 1/2$, and $|r| < 1$, so $S = 3/(1-1/2) = 6$.

   b. The verified result is 6.

10. Test $\sum_{n=1}^{\infty} n/(n+1)$ for convergence.

    Answer: diverges

    a. The terms satisfy $n/(n+1) \to 1 \neq 0$, so the nth-term test proves divergence.

    b. The verified result is diverges.

11. Test $\sum_{n=1}^{\infty} 1/n^{3/2}$.

    Answer: converges

    a. This is a p-series with $p = 3/2 > 1$, so it converges.

    b. The verified result is converges.

12. Test $\sum_{n=1}^{\infty} 1/(n^2+4)$.

    Answer: converges

    a. For $n \geq 1$, $0 < 1/(n^2+4) < 1/n^2$; comparison with the convergent p-series proves convergence.

    b. The verified result is converges.

13. Test $\sum_{n=1}^{\infty} (3n+1)/(n^2+2)$.

    Answer: diverges

    a. With $b_n = 1/n$, $a_n/b_n = n(3n+1)/(n^2+2) \to 3$; limit comparison with the harmonic series gives divergence.

    b. The verified result is diverges.

14. Test $\sum_{n=1}^{\infty} n!/4^n$.

    Answer: diverges

    a. $a_{n+1}/a_n = (n+1)/4$, which eventually exceeds 1, so the positive terms do not approach zero and the series diverges.

    b. The verified result is diverges.

15. Test $\sum_{n=1}^{\infty} (2n/(3n+1))^n$.

    Answer: converges

    a. The root-test limit is $\lim 2n/(3n+1) = 2/3 < 1$, so the series converges absolutely.

    b. The verified result is converges.

16. Test $\sum_{n=1}^{\infty}(-1)^{n-1}/n$.

Answer: converges conditionally

a. $1/n$ decreases to zero, so the alternating-series test gives convergence; $\sum 1/n$ diverges, hence convergence is conditional.

b. The verified result is converges conditionally.

17. Classify $\sum_{n=1}^{\infty}(-1)^{n}/n^{2}$.

Answer: converges absolutely

a. The absolute series $\sum 1/n^2$ is a p-series with $p = 2 > 1$, so the original series converges absolutely.

b. The verified result is converges absolutely.

18. Find R for $\sum_{n=0}^{\infty} x^{n}/5^{n}$.

Answer: $R = 5$

a. This is geometric with ratio $x/5$; convergence requires $|x/5| < 1$, or $|x| < 5$, so $R = 5$.

b. The verified result is $R = 5$.

19. Find the interval for $\sum_{n=1}^{\infty}(x-1)^{n}/n$.

Answer: $[0, 2)$

a. The ratio test gives $|x - 1| < 1$. At $x = 0$, $\sum(-1)^n/n$ converges; at $x = 2$, $\sum 1/n$ diverges, so the interval is $[0, 2)$.

b. The verified result is $[0, 2)$.

20. Write the Maclaurin series for cos x.

Answer: $\sum_{n=0}^{\infty}(-1)^{n}x^{2n}/(2n)!$

a. The derivatives at zero cycle $1, 0, -1, 0$, leaving even powers with alternating signs: $\sum_{n=0}^{\infty}(-1)^{n}x^{2n}/(2n)!$.

b. The verified result is $\sum_{n=0}^{\infty}(-1)^{n}x^{2n}/(2n)!$.

21. Find T_3 for e to the power x at 0.

Answer: $1 + x + x^2/2 + x^3/6$

a. Every derivative of e^x equals 1 at zero, so $T_3 = \sum_{n=0}^{3} x^n/n! = 1 + x + x^2/2 + x^3/6$.

b. The verified result is $1 + x + x^2/2 + x^3/6$.

22. Bound alternating-series error after four terms of $\sum_{n=1}^{\infty}(-1)^{n-1}/n$.

Answer: $\leq 1/5$

a. The terms $1/n$ decrease to zero; after four terms the first omitted magnitude is $1/5$, so $|R_4| \leq 1/5$.

b. The verified result is $\leq 1/5$.

23. For x=t squared,y=t cubed find dy/dx.

Answer: $3t/2$

a. $dx/dt = 2t$ and $dy/dt = 3t^2$, so $dy/dx = (3t^2)/(2t) = 3t/2$ for $t \neq 0$.

b. The verified result is $3t/2$.

24. Convert r=2cos to Cartesian form.

    Answer: $(x-1)^2+y^2=1$

    a. Multiply by r: $r^2=2r\cos\theta$, so $x^2+y^2=2x$, which completes the square to $(x-1)^2+y^2=1$.

    b. The verified result is $(x-1)^2+y^2=1$.

25. Why must power-series endpoints be tested separately?

    Answer: The ratio/root test is inconclusive when $|x-a|=R$.

    a. At $|x-a|=R$, the limiting ratio or root equals 1, so the test is inconclusive and each endpoint series requires its own convergence test.

    b. The verified result is The ratio/root test is inconclusive when $|x-a|=R$..