Chain Rule Worksheet — Worked Answer Key

Twenty-four composition problems from basic powers through nested, product, and quotient combinations.

Revision: 2026-07-23 **Canonical page:** https://bettergrades.net/subjects/math/calculus/worksheets/chain-rule/

1. Differentiate $y = (3x+2)^5$.

   Answer: $y' = 15(3x+2)^4$

   a. Use the power rule on the outer power u^5.
   b. Multiply by the inner derivative 3.
   c. Therefore the result is $y' = 15(3x+2)^4$.

2. Differentiate $y = (5x-1)^4$.

   Answer: $y' = 20(5x-1)^3$

   a. Use the power rule on the outer power u^4.
   b. Multiply by the inner derivative 5.
   c. Therefore the result is $y' = 20(5x-1)^3$.

3. Differentiate $y = (2x+4)^6$.

   Answer: $y' = 12(2x+4)^5$

   a. Use the power rule on the outer power u^6.
   b. Multiply by the inner derivative 2.
   c. Therefore the result is $y' = 12(2x+4)^5$.

4. Differentiate $y = (-3x+2)^3$.

   Answer: $y' = -9(-3x+2)^2$

   a. Use the power rule on the outer power u^3.
   b. Multiply by the inner derivative -3.
   c. Therefore the result is $y' = -9(-3x+2)^2$.

5. Differentiate $y = (7x+1)^2$.

   Answer: $y' = 14(7x+1)^1$

   a. Use the power rule on the outer power u^2.
   b. Multiply by the inner derivative 7.
   c. Therefore the result is $y' = 14(7x+1)^1$.

6. Differentiate $y = (4x-5)^3$.

   Answer: $y' = 12(4x-5)^2$

   a. Use the power rule on the outer power u^3.
   b. Multiply by the inner derivative 4.
   c. Therefore the result is $y' = 12(4x-5)^2$.

7. Differentiate $y = e^{3x^2}$.

   Answer: $y' = 6xe^{3x^2}$

   a. Treat $3x^2$ as the inner function.
   b. Differentiate the outer function and multiply by $6x$.
   c. Therefore the result is $y' = 6xe^{3x^2}$.

8. Differentiate $y = e^{\sin x}$.

   Answer: $y' = e^{\sin x} \cos x$

   a. Treat $\sin x$ as the inner function.
   b. Differentiate the outer function and multiply by $\cos x$.
   c. Therefore the result is $y' = e^{\sin x} \cos x$.

9. Differentiate $y = \ln(5x - 2)$.

   Answer: $y' = \frac{5}{5x-2}$

   a. Treat $5x - 2$ as the inner function.
   b. Differentiate the outer function and multiply by 5.
   c. Therefore the result is $y' = \frac{5}{5x-2}$.

10. Differentiate $y = \ln(x^2 + 4)$.

    Answer: $y' = \frac{2x}{x^2+4}$

    a. Treat $x^2 + 4$ as the inner function.
    b. Differentiate the outer function and multiply by $2x$.
    c. Therefore the result is $y' = \frac{2x}{x^2+4}$.

11. Differentiate $y = \sin(4x^3)$.

    Answer: $y' = 12x^2 \cos(4x^3)$

    a. Identify the outer trigonometric function and preserve its inner input.
    b. Differentiate the inner expression and multiply.
    c. Therefore the result is $y' = 12x^2 \cos(4x^3)$.

12. Differentiate $y = \cos(2x - 1)$.

    Answer: $y' = -2\sin(2x - 1)$

    a. Identify the outer trigonometric function and preserve its inner input.
    b. Differentiate the inner expression and multiply.
    c. Therefore the result is $y' = -2\sin(2x - 1)$.

13. Differentiate $y = \tan(x^2)$.

    Answer: $y' = 2x\sec^2(x^2)$

    a. Identify the outer trigonometric function and preserve its inner input.
    b. Differentiate the inner expression and multiply.

c. Therefore the result is $y' = 2x\sec^2(x^2)$.

14. Differentiate $y = \sin^3 x$.

    Answer: $y' = 3\sin^2 x\cos x$

    a. Identify the outer trigonometric function and preserve its inner input.
    b. Differentiate the inner expression and multiply.
    c. Therefore the result is $y' = 3\sin^2 x\cos x$.

15. Differentiate $y = (1 + (2x-3)^2)^4$.

    Answer: $y' = 16(2x-3)(1+(2x-3)^2)^3$

    a. Write the composition as nested layers from outside to inside.
    b. Differentiate each layer once and multiply the factors.
    c. Therefore the result is $y' = 16(2x-3)(1+(2x-3)^2)^3$.

16. Differentiate $y = e^{\sqrt{1+x^2}}$.

    Answer: $y' = \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$

    a. Write the composition as nested layers from outside to inside.
    b. Differentiate each layer once and multiply the factors.
    c. Therefore the result is $y' = \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$.

17. Differentiate $y = \ln(1+\sin^2 x)$.

    Answer: $y' = \frac{2\sin x\cos x}{1+\sin^2 x}$

    a. Write the composition as nested layers from outside to inside.
    b. Differentiate each layer once and multiply the factors.
    c. Therefore the result is $y' = \frac{2\sin x\cos x}{1+\sin^2 x}$.

18. Differentiate $y = x^2 e^{3x}$.

    Answer: $y' = e^{3x}(2x+3x^2)$

    a. Apply the product plus chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $e^{3x}(2x+3x^2)$.
    d. Therefore the result is $y' = e^{3x}(2x+3x^2)$.

19. Differentiate $y = x\sin(x^2)$.

    Answer: $y' = \sin(x^2) + 2x^2\cos(x^2)$

    a. Apply the product plus chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $\sin(x^2) + 2x^2\cos(x^2)$.
    d. Therefore the result is $y' = \sin(x^2) + 2x^2\cos(x^2)$.

20. Differentiate $y = \frac{\ln x}{x^2}$.

    Answer: $y' = \frac{1-2\ln x}{x^3}$

    a. Apply the quotient plus chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $\frac{1-2\ln x}{x^3}$.
    d. Therefore the result is $y' = \frac{1-2\ln x}{x^3}$.

21. Differentiate $y = \frac{e^x}{1+x^2}$.

    Answer: $y' = \frac{e^x(1+x^2-2x)}{(1+x^2)^2}$

    a. Apply the quotient rule structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $\frac{e^x(1+x^2-2x)}{(1+x^2)^2}$.
    d. Therefore the result is $y' = \frac{e^x(1+x^2-2x)}{(1+x^2)^2}$.

22. Differentiate $y = (x^2+1)^3(x-2)$.

    Answer: $y' = (x^2+1)^2(7x^2-12x+1)$

    a. Apply the product plus chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $(x^2+1)^2(7x^2-12x+1)$.
    d. Therefore the result is $y' = (x^2+1)^2(7x^2-12x+1)$.

23. Differentiate $y = \frac{\sin(2x)}{x}$.

    Answer: $y' = \frac{2x\cos(2x)-\sin(2x)}{x^2}$

    a. Apply the quotient plus chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $\frac{2x\cos(2x)-\sin(2x)}{x^2}$.
    d. Therefore the result is $y' = \frac{2x\cos(2x)-\sin(2x)}{x^2}$.

24. Differentiate $y = \sqrt{1+e^{2x}}$.

    Answer: $y' = \frac{e^{2x}}{\sqrt{1+e^{2x}}}$

    a. Apply the nested chain structure before simplifying.
    b. Retain every factor contributed by an inner derivative.
    c. A factored final form is $\frac{e^{2x}}{\sqrt{1+e^{2x}}}$.
    d. Therefore the result is $y' = \frac{e^{2x}}{\sqrt{1+e^{2x}}}$.