Evaluating Limits Worksheet — Worked Answer Key

Twenty-four limits that progress from substitution to algebraic, one-sided, and infinite behavior.

Revision: 2026-07-23 **Canonical page:** https://bettergrades.net/subjects/math/calculus/worksheets/evaluating-limits/

1. Evaluate $\lim_{x\to 2}(3x^2 - 5x + 4)$.

   Answer: 6

   a. The expression is continuous at the target input 2.
   b. Substitute the target and simplify to obtain 6.

2. Evaluate $\lim_{x\to -2}(x^3 + 4x)$.

   Answer: -16

   a. The expression is continuous at the target input -2.
   b. Substitute the target and simplify to obtain -16.

3. Evaluate $\lim_{t\to 3} \frac{t^2+1}{t+2}$.

   Answer: 2

   a. The expression is continuous at the target input 3.
   b. Substitute the target and simplify to obtain 2.

4. Evaluate $\lim_{h\to 0}(7 - 4h + h^2)$.

   Answer: 7

   a. The expression is continuous at the target input 0.
   b. Substitute the target and simplify to obtain 7.

5. Evaluate $\lim_{u\to 1} \sqrt{u+8}$.

   Answer: 3

   a. The expression is continuous at the target input 1.
   b. Substitute the target and simplify to obtain 3.

6. Evaluate $\lim_{x\to 4} \frac{x+5}{\sqrt{x}}$.

   Answer: $\frac{9}{2}$

   a. The expression is continuous at the target input 4.
   b. Substitute the target and simplify to obtain $\frac{9}{2}$.

7. Evaluate $\lim_{x\to 3} \frac{x^2-9}{x-3}$.

   Answer: 6

   a. Factor the vanishing numerator as $(x-3)(x+3)$.
   b. Cancel the common factor only for nearby inputs, then substitute the target.
   c. The simplified expression approaches 6.

8. Evaluate $\lim_{x \to -4} \frac{x^2-16}{x+4}$.

   Answer: -8

   a. Factor the vanishing numerator as $(x+4)(x-4)$.
   b. Cancel the common factor only for nearby inputs, then substitute the target.
   c. The simplified expression approaches -8.

9. Evaluate $\lim_{x \to 2} \frac{x^3-8}{x-2}$.

   Answer: 12

   a. Factor the vanishing numerator as $(x-2)(x^2+2x+4)$.
   b. Cancel the common factor only for nearby inputs, then substitute the target.
   c. The simplified expression approaches 12.

10. Evaluate $\lim_{x \to 1} \frac{x^2+x-2}{x-1}$.

    Answer: 3

    a. Factor the vanishing numerator as $(x-1)(x+2)$.
    b. Cancel the common factor only for nearby inputs, then substitute the target.
    c. The simplified expression approaches 3.

11. Evaluate $\lim_{y \to 5} \frac{y^2-7y+10}{y-5}$.

    Answer: 3

    a. Factor the vanishing numerator as $(y-5)(y-2)$.
    b. Cancel the common factor only for nearby inputs, then substitute the target.
    c. The simplified expression approaches 3.

12. Evaluate $\lim_{z \to -1} \frac{z^3+1}{z+1}$.

    Answer: 3

    a. Factor the vanishing numerator as $(z+1)(z^2-z+1)$.
    b. Cancel the common factor only for nearby inputs, then substitute the target.
    c. The simplified expression approaches 3.

13. Evaluate $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$.

    Answer: $\frac{1}{6}$

    a. Multiply numerator and denominator by the conjugate $\sqrt{x}+3$.
    b. Use the difference-of-squares identity and cancel the factor that tends to zero.
    c. Substitution in the simplified expression gives $\frac{1}{6}$.

14. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$.

    Answer: $\frac{1}{2}$

    a. Multiply numerator and denominator by the conjugate $\sqrt{1+x}+1$.
    b. Use the difference-of-squares identity and cancel the factor that tends to zero.

c. Substitution in the simplified expression gives $\frac{1}{2}$.

15. Evaluate $\lim_{t\to 4} \frac{t-4}{\sqrt{t}-2}$.

    Answer: 4

    a. Multiply numerator and denominator by the conjugate $\sqrt{t}+2$.
    b. Use the difference-of-squares identity and cancel the factor that tends to zero.
    c. Substitution in the simplified expression gives 4.

16. Evaluate $\lim_{u\to 0} \frac{u}{\sqrt{9+u}-3}$.

    Answer: 6

    a. Multiply numerator and denominator by the conjugate $\sqrt{9+u}+3$.
    b. Use the difference-of-squares identity and cancel the factor that tends to zero.
    c. Substitution in the simplified expression gives 6.

17. For $f(x) = \begin{cases} x+2, & x<1 \\ 4-x, & x\geq 1 \end{cases}$, evaluate the limit as $x \to 1^-$.

    Answer: 3

    a. Approaching from the left selects the branch $f(x) = x+2$.
    b. As $x \to 1^-$, $x+2 \to 3$; the value assigned by the other branch at the endpoint does not control this one-sided limit.
    c. Therefore the result is 3.

18. For $f(x) = \begin{cases} 2x, & x<0 \\ x^2+1, & x\geq 0 \end{cases}$, evaluate the limit as $x \to 0^+$.

    Answer: 1

    a. Approaching from the right selects the branch $f(x) = x^2+1$.
    b. As $x \to 0^+$, $x^2+1 \to 1$.
    c. Therefore the result is 1.

19. For $g(x) = \frac{|x|}{x}$, evaluate the limit as $x \to 0^-$.

    Answer: -1

    a. For $x<0$, $|x| = -x$, so $g(x) = (-x)/x = -1$.
    b. Therefore the values remain -1 as $x \to 0^-$.

20. For $h(x) = \begin{cases} x^2, & x\leq 2 \\ 6-x, & x>2 \end{cases}$, evaluate the limit as $x \to 2$.

    Answer: 4

    a. As $x \to 2^-$, the branch x^2 approaches 4.
    b. As $x \to 2^+$, the branch $6-x$ approaches 4.
    c. Because both one-sided limits equal 4, the two-sided limit is 4.

21. Evaluate $\lim_{x\to\infty} \frac{5x^2-1}{2x^2+3}$.

    Answer: $\frac{5}{2}$

    a. Identify the dominant behavior: equal degrees.
    b. Divide by the highest relevant power or use the sign of the vanishing factor.
    c. The limit is $\frac{5}{2}$.

22. Evaluate $\lim_{x\to\infty} \frac{3x+7}{x^2+1}$.

    Answer: 0

    a. Identify the dominant behavior: denominator degree is larger.
    b. Divide by the highest relevant power or use the sign of the vanishing factor.
    c. The limit is 0.

23. Evaluate $\lim_{x\to-\infty} \frac{4x^3+x}{2x^3-5}$.

    Answer: 2

    a. Identify the dominant behavior: equal degrees.
    b. Divide by the highest relevant power or use the sign of the vanishing factor.
    c. The limit is 2.

24. Evaluate $\lim_{x\to 2^+} \frac{1}{x-2}$.

    Answer: $+\infty$

    a. Identify the dominant behavior: positive denominator approaching zero.
    b. Divide by the highest relevant power or use the sign of the vanishing factor.
    c. The limit is $+\infty$.

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