Integration by Parts Worksheet — Worked Answer Key

Twenty problems covering polynomial products, logarithms, inverse trigonometric functions, definite integrals, and cyclic cases.

Revision: 2026-07-23 **Canonical page:** https://bettergrades.net/subjects/math/calculus/worksheets/integration-by-parts/

1. Evaluate $\int xe^x\,dx$.

   Answer: $e^x(x-1)+C$

   a. Take $u=x$ and $dv=e^x dx$, so $du=dx$ and $v=e^x$.
   b. Then $\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C = e^x(x-1)+C$.
   c. Therefore the result is $e^x(x-1)+C$.

2. Evaluate $\int x^2e^x\,dx$.

   Answer: $e^x(x^2-2x+2)+C$

   a. Take $u=x^2$ and $dv=e^x dx$: $I = x^2e^x - 2\int xe^x dx$.
   b. Using $\int xe^x dx = e^x(x-1)$, $I=e^x(x^2-2x+2)+C$.
   c. Therefore the result is $e^x(x^2-2x+2)+C$.

3. Evaluate $\int x^3e^x\,dx$.

   Answer: $e^x(x^3-3x^2+6x-6)+C$

   a. Take $u=x^3$ and $dv=e^x dx$: $I=x^3e^x-3\int x^2e^x dx$.
   b. Substitute $\int x^2e^x dx = e^x(x^2-2x+2)$ and collect terms to get $e^x(x^3-3x^2+6x-6)+C$.

4. Evaluate $\int xe^{2x}\,dx$.

   Answer: $e^{2x}(2x-1)/4+C$

   a. Take $u=x$ and $dv=e^{2x}dx$, so $du=dx$ and $v=e^{2x}/2$.
   b. Thus $I=xe^{2x}/2-\int e^{2x}/2\,dx = xe^{2x}/2 - e^{2x}/4 + C = e^{2x}(2x-1)/4+C$.
   c. Therefore the result is $e^{2x}(2x-1)/4+C$.

5. Evaluate $\int x\sin x\,dx$.

   Answer: $-x\cos x+\sin x+C$

   a. Take $u=x$ and $dv=\sin x\,dx$, so $du=dx$ and $v=-\cos x$.
   b. Then $I=-x\cos x+\int\cos x\,dx = -x\cos x+\sin x+C$.
   c. Therefore the result is $-x\cos x+\sin x+C$.

6. Evaluate $\int x\cos x\,dx$.

   Answer: $x\sin x+\cos x+C$

   a. Take $u=x$ and $dv=\cos x\,dx$, so $du=dx$ and $v=\sin x$.
   b. Then $I=x\sin x-\int\sin x\,dx = x\sin x+\cos x+C$.
   c. Therefore the result is $x\sin x+\cos x+C$.

7. Evaluate $\int x^2 \sin x \, dx$.

   Answer: $-x^2 \cos x + 2x \sin x + 2 \cos x + C$

   a. Take $u = x^2$ and $dv = \sin x \, dx$: $I = -x^2 \cos x + 2 \int x \cos x \, dx$.
   b. Since $\int x \cos x \, dx = x \sin x + \cos x$, $I = -x^2 \cos x + 2x \sin x + 2 \cos x + C$.
   c. Therefore the result is $-x^2 \cos x + 2x \sin x + 2 \cos x + C$.

8. Evaluate $\int x^2 \cos x \, dx$.

   Answer: $x^2 \sin x + 2x \cos x - 2 \sin x + C$

   a. Take $u = x^2$ and $dv = \cos x \, dx$: $I = x^2 \sin x - 2 \int x \sin x \, dx$.
   b. Since $\int x \sin x \, dx = -x \cos x + \sin x$, $I = x^2 \sin x + 2x \cos x - 2 \sin x + C$.
   c. Therefore the result is $x^2 \sin x + 2x \cos x - 2 \sin x + C$.

9. Evaluate $\int \ln x \, dx$.

   Answer: $x \ln x - x + C$

   a. Write the integrand as $(\ln x)(1)$; take $u = \ln x$ and $dv = dx$, so $du = dx/x$ and $v = x$.
   b. Then $I = x \ln x - \int 1 \, dx = x \ln x - x + C$, for $x > 0$.
   c. Therefore the result is $x \ln x - x + C$.

10. Evaluate $\int x \ln x \, dx$.

    Answer: $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

    a. Take $u = \ln x$ and $dv = x \, dx$, so $du = dx/x$ and $v = x^2/2$.
    b. Then $I = (x^2/2) \ln x - \frac{1}{2} \int x \, dx = (x^2/2) \ln x - x^2/4 + C$, for $x > 0$.
    c. Therefore the result is $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$.

11. Evaluate $\int (\ln x)^2 \, dx$.

    Answer: $x[(\ln x)^2 - 2 \ln x + 2] + C$

    a. Take $u = (\ln x)^2$ and $dv = dx$: $I = x(\ln x)^2 - 2 \int \ln x \, dx$.
    b. Substitute $\int \ln x \, dx = x \ln x - x$ to obtain $x[(\ln x)^2 - 2 \ln x + 2] + C$, for $x > 0$.

12. Evaluate $\int \arctan x \, dx$.

    Answer: $x \arctan x - \frac{1}{2} \ln(1 + x^2) + C$

    a. Take $u = \arctan x$ and $dv = dx$, so $du = dx/(1 + x^2)$ and $v = x$.
    b. Then $I = x \arctan x - \int x/(1 + x^2) \, dx = x \arctan x - \frac{1}{2} \ln(1 + x^2) + C$.
    c. Therefore the result is $x \arctan x - \frac{1}{2} \ln(1 + x^2) + C$.

13. Evaluate $\int \arcsin x \, dx$.

    Answer: $x \arcsin x + \sqrt{1 - x^2} + C$

    a. Take $u = \arcsin x$ and $dv = dx$, so $du = dx/\sqrt{1 - x^2}$ and $v = x$.
    b. Because $\int x/\sqrt{1 - x^2} \, dx = -\sqrt{1 - x^2}$, $I = x \arcsin x + \sqrt{1 - x^2} + C$ on $(-1, 1)$.
    c. Therefore the result is $x \arcsin x + \sqrt{1 - x^2} + C$.

14. Evaluate $\int_0^1 xe^x\,dx$.

    Answer: 1

    a. Using $u = x$ and $dv = e^x dx$, an antiderivative is $e^x(x-1)$.
    b. Evaluate the bounds: $[e^x(x-1)]_0^1 = 0 - (-1) = 1$.
    c. Therefore the result is 1.

15. Evaluate $\int_0^\pi x\sin x\,dx$.

    Answer: π

    a. Using $u = x$ and $dv = \sin x\,dx$, an antiderivative is $-x\cos x + \sin x$.
    b. Evaluate the bounds: $[-x\cos x + \sin x]_0^\pi = \pi - 0 = \pi$.
    c. Therefore the result is π.

16. Evaluate $\int e^x \cos x\,dx$.

    Answer: $\frac{e^x}{2}(\sin x + \cos x) + C$

    a. Let $I = \int e^x \cos x\,dx$. Two integrations by parts give $I = e^x\cos x + e^x \sin x - I$.
    b. Therefore $2I = e^x(\sin x + \cos x)$, so $I = \frac{e^x}{2}(\sin x + \cos x) + C$.
    c. Therefore the result is $\frac{e^x}{2}(\sin x + \cos x) + C$.

17. Evaluate $\int e^x \sin x\,dx$.

    Answer: $\frac{e^x}{2}(\sin x - \cos x) + C$

    a. Let $I = \int e^x \sin x\,dx$. Two integrations by parts give $I = e^x\sin x - e^x \cos x - I$.
    b. Therefore $2I = e^x(\sin x - \cos x)$, so $I = \frac{e^x}{2}(\sin x - \cos x) + C$.
    c. Therefore the result is $\frac{e^x}{2}(\sin x - \cos x) + C$.

18. Evaluate $\int x^4 e^x\,dx$.

    Answer: $e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C$

    a. Repeatedly take the polynomial as u: $I = x^4e^x - 4\int x^3e^x dx$.
    b. Substituting the three prior reductions gives $I = e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C$.
    c. Therefore the result is $e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C$.

19. Evaluate $\int x^2 \ln x\,dx$.

    Answer: $\frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

    a. Take $u = \ln x$ and $dv = x^2dx$, so $du = dx/x$ and $v = x^3/3$.
    b. Then $I = (x^3/3)\ln x - \frac{1}{3}\int x^2dx = (x^3/3)\ln x - x^3/9 + C$, for $x > 0$.
    c. Therefore the result is $\frac{x^3}{3}\ln x - \frac{x^3}{9} + C$.

20. Evaluate $\int_1^e \ln x\,dx$.

    Answer: 1

    a. Using $u = \ln x$ and $dv = dx$, an antiderivative is $x\ln x - x$.
    b. Evaluate the bounds: $[x\ln x - x]_1^e = 0 - (-1) = 1$.
    c. Therefore the result is 1.