Optimization Worksheet — Worked Answer Key

Sixteen substantial modeling problems with constraints, domains, critical points, and verification.

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1. Squares of side x are cut from a 12 by 12 sheet. Maximize the open box volume.

   Answer: $x = 2,\ V_{\max} = 128$

   a. V=x(12-2x) to the power 2 on 0<x<6; V'=12(x-2)(x-6), so the interior maximum is x=2.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $x = 2,\ V_{\max} = 128$.

2. Squares of side x are cut from a 20 by 12 sheet. Maximize volume.

   Answer: $x = \frac{16-\sqrt{76}}{3} \approx 2.43$

   a. V=x(20-2x)(12-2x). Solve V'=12x to the power 2-128x+240=0 and retain the feasible critical point.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $x = \frac{16-\sqrt{76}}{3} \approx 2.43$.

3. Use 240 m of fence for three sides of a riverside rectangle. Maximize area.

   Answer: $x = 60,\ y = 120,\ A = 7200$

   a. With 2x+y=240, A=x(240-2x); A'=240-4x=0.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $x = 60,\ y = 120,\ A = 7200$.

4. A 600 m rectangle has one divider parallel to its width. Maximize area.

   Answer: $w = 100, \ell = 150,\ A = 15000$

   a. The constraint is 3w+2l=600. Substitute l=(600-3w)/2 into A=wl.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $w = 100, \ell = 150,\ A = 15000$.

5. A poster has area 384 cm squared with 2 cm side margins and 3 cm top and bottom margins. Minimize total paper area.

   Answer: $\textit{printed width} = 16,\ \textit{printed height} = 24$

   a. For printed width x, paper area is (x+4)(384/x+6). Set its derivative to zero.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $\textit{printed width} = 16,\ \textit{printed height} = 24$.

6. Find the closed cylinder of volume 250pi with minimum surface area.

   Answer: $r = 5,\ h = 10$

   a. Use h=250/r to the power 2 in S=2pi r to the power 2+2pi rh; solve S'=0.

b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
c. Therefore the result is $r = 5,\ h = 10$.

7. Find the point on y=x squared closest to (0,3).

   Answer: $(\pm\sqrt{5/2}, 5/2)$

   a. Minimize D to the power 2=x to the power 2+(x to the power 2-3) to the power 2. Critical points satisfy 2x(2x to the power 2-5)=0; compare values.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $(\pm\sqrt{5/2}, 5/2)$.

8. Maximize the area of a rectangle inscribed under y=square root of (25-x to the power 2).

   Answer: $width = 5\sqrt{2},\ height = 5/\sqrt{2},\ A = 25$

   a. A=2xsquare root of (25-x to the power 2); maximize A squared=4x squared(25-x squared).
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $width = 5\sqrt{2},\ height = 5/\sqrt{2},\ A = 25$.

9. Cut 20 m of wire into a square and a circle to minimize total area.

   Answer: $square\ length = \frac{80}{4+\pi},\ circle\ length = \frac{20\pi}{4+\pi}$

   a. Let x go to the square: A=x squared/16+(20-x) squared/(4pi). Solve A'=0.
   b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
   c. Therefore the result is $square\ length = \frac{80}{4+\pi},\ circle\ length = \frac{20\pi}{4+\pi}$.

10. Demand is p=80-2q. Maximize revenue.

    Answer: $q = 20,\ p = 40,\ R = 800$

    a. R=q(80-2q); R'=80-4q=0 and R"<0.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $q = 20,\ p = 40,\ R = 800$.

11. Minimize average cost C(q)/q when C(q)=q squared+100q+2500.

    Answer: $q = 50$

    a. Average cost is q+100+2500/q. Set 1-2500/q squared=0.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $q = 50$.

12. A 10 ft ladder rests against a wall. Maximize the area of the right triangle it forms.

    Answer: $x = y = 5\sqrt{2},\ A = 25$

    a. With x squared+y squared=100, maximize A=xy/2; symmetry or differentiation gives x=y.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $x = y = 5\sqrt{2},\ A = 25$.

13. A cone has volume 72pi. Under the material model S=pi r squared+pi rh, minimize S.

    Answer: $r = \sqrt[3]{108} = 3\sqrt[3]{4},\ h = 2\sqrt[3]{108} = 6\sqrt[3]{4}$

    a. Use h=216/r squared, so S=pi r squared+216pi/r. Then S'=2pi r-216pi/r squared=0 gives r cubed=108 and h=216/r squared=2r.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $r = \sqrt[3]{108} = 3\sqrt[3]{4},\ h = 2\sqrt[3]{108} = 6\sqrt[3]{4}$.

14. A Norman window has perimeter 20. Maximize area.

    Answer: $r = \frac{20}{4+\pi}$

    a. Use width 2r and rectangular height h. The perimeter gives h=(20-(2+pi)r)/2; maximize 2rh+pi r squared/2.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $r = \frac{20}{4+\pi}$.

15. A boat 3 km offshore must reach a town 8 km downshore. Row at 3 km/h and walk at 5 km/h. Minimize time.

    Answer: $land\ about\ 5.75\ km\ before\ town$

    a. Let x be the downshore rowing distance. T=square root of (x squared+9)/3+(8-x)/5; solve T'=0.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $land\ about\ 5.75\ km\ before\ town$.

16. Maximize f(x)=x(6-x) on the closed interval [1,5].

    Answer: $x = 3,\ f(3) = 9$

    a. Check the critical point x=3 and both endpoints; the largest listed value is 9.
    b. Check feasibility, endpoints, and the sign or second derivative before declaring the optimum.
    c. Therefore the result is $x = 3,\ f(3) = 9$.