Taylor Series Worksheet — Worked Answer Key

Twenty problems on known series, transformations, coefficients, intervals, approximations, and error.

Revision: 2026-07-23 **Canonical page:** https://bettergrades.net/subjects/math/calculus/worksheets/taylor-series/

1. Find the degree-3 Maclaurin polynomial for e to the power x.

   Answer: $1 + x + x^2/2 + x^3/6$

   a. Use $e^x = \sum_{n=0}^{\infty} x^n/n!$.
   b. Retaining degrees 0 through 3 gives $1 + x + x^2/2 + x^3/6$.

2. Find the degree-5 Maclaurin polynomial for sin x.

   Answer: $x - x^3/6 + x^5/120$

   a. Use $\sin x = \sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)!$.
   b. Retaining terms through degree 5 gives $x - x^3/6 + x^5/120$.

3. Find the degree-4 Maclaurin polynomial for cos x.

   Answer: $1 - x^2/2 + x^4/24$

   a. Use $\cos x = \sum_{n=0}^{\infty}(-1)^n x^{2n}/(2n)!$.
   b. Retaining terms through degree 4 gives $1 - x^2/2 + x^4/24$.

4. Write the first four nonzero terms of ln(1+x).

   Answer: $x - x^2/2 + x^3/3 - x^4/4$

   a. Integrate the geometric series for $1/(1 + x)$ term by term.
   b. The first four nonzero terms are $x - x^2/2 + x^3/3 - x^4/4$.

5. Expand 1/(1-x) as a power series.

   Answer: $\sum_{n=0}^{\infty} x^n,\ |x| < 1$

   a. The geometric identity $1/(1 - r) = \sum_{n=0}^{\infty} r^n$ requires $|r| < 1$.
   b. Set $r = x$ to obtain $\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.
   c. Therefore the result is $\sum_{n=0}^{\infty} x^n,\ |x| < 1$.

6. Expand 1/(1+x to the power 2) as a power series.

   Answer: $\sum_{n=0}^{\infty}(-1)^n x^{2n},\ |x| < 1$

   a. Set $r = -x^2$ in the geometric series.
   b. This gives $\sum_{n=0}^{\infty}(-1)^n x^{2n}$, with $|x| < 1$.
   c. Therefore the result is $\sum_{n=0}^{\infty}(-1)^n x^{2n},\ |x| < 1$.

7. Expand 1/(1-3x) as a power series.

   Answer: $\sum_{n=0}^{\infty} 3^n x^n,\ |x| < 1/3$

   a. Set $r = 3x$ in the geometric series.

b. The condition $|3x| < 1$ gives $|x| < 1/3$.
c. Therefore the result is $\sum_{n=0}^{\infty} 3^n x^n,\ |x| < 1/3$.

8. Find a power series for 1/(1-x) to the power 2.

   Answer: $\sum_{n=1}^{\infty} nx^{n-1},\ |x| < 1$

   a. Differentiate $1/(1-x) = \sum_{n=0}^{\infty} x^n$ term by term.
   b. This gives $1/(1-x)^2 = \sum_{n=1}^{\infty} nx^{n-1}$ for $|x| < 1$.
   c. Therefore the result is $\sum_{n=1}^{\infty} nx^{n-1},\ |x| < 1$.

9. Find a power series for arctan x by integrating 1/(1+x squared), and state its interval of convergence.

   Answer: $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1),\ -1 \le x \le 1$

   a. Integrate $1/(1+x^2) = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ from 0 to x.
   b. The result is $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)$; direct endpoint tests include both $x = -1$ and $x = 1$.
   c. Therefore the result is $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1),\ -1 \le x \le 1$.

10. Find the degree-2 Taylor polynomial for ln x centered at 1.

    Answer: $(x-1) - (x-1)^2/2$

    a. For $f(x) = \ln x$, $f(1) = 0$, $f'(1) = 1$, and $f''(1) = -1$.
    b. Substitution in the degree-2 Taylor formula gives $(x-1) - (x-1)^2/2$.

11. Write the Taylor series for e to the power x centered at a.

    Answer: $e^a \sum_{n=0}^{\infty}(x-a)^n/n!$

    a. Every derivative of e^x equals e^x, so $f^{(n)}(a) = e^a$.
    b. The Taylor formula gives $e^a \sum_{n=0}^{\infty}(x-a)^n/n!$, convergent for every real x.

12. If f to the power (n)(0)=2 to the power n, find the Maclaurin series.

    Answer: $\sum_{n=0}^{\infty} 2^n x^n/n! = e^{2x}$

    a. The Maclaurin coefficient is $f^{(n)}(0)/n! = 2^n/n!$.
    b. Thus the series is $\sum_{n=0}^{\infty} 2^n x^n/n! = e^{2x}$.

13. Use T_3 for e to the power x to approximate e to the power (0.1).

    Answer: $1.105166\overline{6}$

    a. Evaluate $T_3(0.1) = 1 + 0.1 + 0.1^2/2 + 0.1^3/6$.
    b. The result is $1.105166\overline{6}$.

14. Use x-x cubed/6 to approximate sin(0.2).

    Answer: $0.198666\overline{6}$

    a. Evaluate $0.2 - (0.2)^3/6$.
    b. The result is $0.198666\overline{6}$.

15. Find the radius of $\sum_{n=1}^{\infty} nx^n/4^n$.

    Answer: $R = 4$

    a. The ratio of successive absolute terms approaches $|x|/4$.
    b. Convergence requires $|x| < 4$, so $R = 4$.

16. Find the interval of convergence of $\sum_{n=1}^{\infty} x^n/n$.

    Answer: $[-1, 1)$

    a. The radius is 1. At $x = 1$ the harmonic series diverges; at $x = -1$ the alternating harmonic series converges.
    b. Therefore the interval is $[-1, 1)$.

17. Find the interval of convergence of $\sum_{n=1}^{\infty}(x-2)^n/(n3^n)$.

    Answer: $[-1, 5)$

    a. The ratio test gives $|x-2| < 3$, hence $-1 < x < 5$.
    b. At $x = -1$ the series is alternating harmonic and converges; at $x = 5$ it is harmonic and diverges, giving $[-1, 5)$.

18. Find R for $\sum_{n=0}^{\infty} n!(x+1)^n/(2n)!$.

    Answer: $R = \infty$

    a. The coefficient ratio satisfies $|a_n/a_{n+1}| = (2n+2)(2n+1)/(n+1) = 2(2n+1)$.
    b. This tends to infinity, so $R = \infty$.

19. Bound the error of the degree-3 Maclaurin approximation to e to the power (0.2).

    Answer: $|R_3| \le e^{0.2}(0.2)^4/4! < 0.000082$

    a. Taylor's theorem gives $|R_3| \le e^{0.2}(0.2)^4/4!$ on $[0, 0.2]$.
    b. The bound is approximately $0.00008143 < 0.000082$.
    c. Therefore the result is $|R_3| \le e^{0.2}(0.2)^4/4! < 0.000082$.

20. How many terms of the alternating arctan series ensure error below 0.001 at x=1/2?

    Answer: 4 terms

    a. After N terms, the alternating-series error is at most $(1/2)^{2N+1}/(2N+1)$.
    b. Three terms give about $0.001116 > 0.001$, while four give about $0.000217 < 0.001$, so four terms are required.
    c. Therefore the result is 4 terms.